Destabilizing Search Technology*

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Abstract

Modern search technologies enable workers to monitor—and thus quickly apply to—newly posted jobs. I study how monitoring technologies affect search decisions and equilibrium labor market dynamics. The central insight is that monitoring technologies give rise to a matching process in which workers who choose to actively monitor the arrival of vacancies directly crowd out those who do not, all of whom compete for a depleted stock of remaining jobs. This dynamic implies increasing returns to monitoring and thus potentially destabilizing multiplicity. I first illustrate the source of multiplicity in a stylized monitoring game in which unemployed workers compete for vacancies, and then embed the game in a quantitative dynamic model of the labor market. With a plausibly elastic job creation process (i.e., away from the free-entry limit), the quantitative model provides: (i) a theory of belief-driven fluctuations in labor supply that can permanently alter the path of the economy, (ii) a mechanism through which transitory demand shocks can permanently affect labor supply, and (iii) a parsimonious account of the recovery from the Great Recession, during which an historically tight labor market coexisted with weak wage growth—observations difficult to reconcile with traditional models. I document two facts that are suggestive of the model and its implications.

Keywords: Search; matching; unemployment; multiple equilibria
JEL Classification: E24, J64, E71

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1 Introduction

Since the turn of the century, online job search has become ubiquitous in the U.S. labor market. Between 2000 and 2010, the share of unemployed workers in the U.S. using the internet to search for work increased from 25% to 75% (Faberman and Kudlyak, 2016; Kuhn and Mansour, 2013), while, as of 2013, 70% of all job openings in the U.S. were posted online (Carnevale et al., 2014). Understanding the consequences of this shift to online search requires understanding how various online search technologies affect the matching process and thus equilibrium labor market dynamics. Motivated by these questions, a growing literature has begun studying the channels through which online search technologies operate. Recent contributions to this literature include work considering how information provision to workers can expand the scope of job search (Belot et al., 2019), how declining search costs can make workers more selective (Menzio and Martellini, 2020), and how online recruitment by firms can improve screening of applicants (Pries and Rogerson, 2022).

This paper studies a feature of online search technologies not previously explored in the literature: Monitoring technologies that enable job seekers to find and apply to jobs soon after they are posted. A number of technologies facilitate monitoring: Personal computers and broadband connections enable job seekers to easily and frequently check for new listings, job search engines sort listings by the date on which they were posted thus facilitating finding those posted most recently, and matching platforms offer "job alerts" to notify job seekers of new listings that match their profiles.[1] By reducing the fixed costs of frequently checking for jobs, such technologies allow workers to effectively monitor the arrival of new listings, thus offering a first-mover advantage to those who do so but leaving few jobs remaining for those who do not. This aspect of the search decision—made salient by the introduction of monitoring technologies—is absent from traditional models of search. This paper's contribution is to study the implications of viewing search as a monitoring decision.[2] There are two main results: (i) Monitoring technologies can give rise to multiple equilibria, and (ii) such multiplicity can destabilize the economy.

Table 1: Monitoring game

Worker i \ Everybody else	Monitor	Wait
Monitor	θw	w
Wait	κ	$\kappa + \theta w$

The first main result—that monitoring technologies can lead to multiple equilibria—can be understood in a stylized one-period game in which u unemployed workers compete for $v < u$ vacant jobs. I operationalize the notion of search as a monitoring decision by assuming that workers choose between matching in the morning (when jobs are first posted) and enjoying a day of leisure before

[1]Job alerts are offered by virtually all major job search platforms. For example, Monster.com tells searchers that they can "Get relevant jobs matching your profile and criteria straight in your inbox with our Free Job Alerts" and visitors to Indeed.com are greeted by a pop-up inviting them to enter their email address to "Be the first to see new jobs in [your location]."

[2]Davis and Samaniego de la Parra (2020) document pervasive "application bunching" (a phenomenon in which vacancies receive a flood of applications immediately after posting) in second-by-second data linking applications to job postings from DHI Group, Inc. This is strongly suggestive of workers monitoring the arrival of new vacancies.

matching with remaining vacancies in the evening. This is a simple way of capturing that monitoring entails frequent search, thus giving searchers a first-mover advantage over those who wait. Suppose, for now, that matching is frictionless in both the morning and the evening, and let w denote the value of finding a job, $\theta \equiv v/u$ the probability of finding a job if a worker applies to jobs at the same time as other workers (i.e., either in the morning by monitoring or in the evening by waiting), and κ the value of leisure. Table 1 depicts this game, with a particular worker's choice represented by the rows and the (symmetric) choices of all other workers depicted by the columns. Potential equilibria are represented by diagonal entries in the matrix.

Suppose all other workers wait to search in the evening. If an individual worker does the same, she enjoys a day of leisure and competes with other workers for jobs in the evening, yielding an expected payoff of $\kappa + \theta w$. On the other hand, if she decides to apply for jobs as soon as they are available in the morning, she gets a first-mover advantage and is thus assured to find a job, but forgoes leisure, giving her a payoff of w. Now, instead, suppose all other workers monitor and thus apply to jobs as soon as they become available in the morning. If our unemployed worker does the same, she now must compete with these workers in the morning and receives no leisure, yielding an expected payoff of θw. If instead she enjoys leisure and waits, there are no remaining jobs left in the evening, so she only receives κ.

It is straightforward to show that this game has two symmetric equilibria if $\kappa \in [(1-\theta)w, \theta w]$. Furthermore, this condition is increasingly likely to hold in a tight labor market: As $\theta \to 1$, $\kappa < w$ is sufficient for multiplicity. Multiplicity can arise because the availability of monitoring technologies leads to increasing returns to early search: If a worker believes that others are waiting until the evening to search, then forgoing leisure to actively monitor the arrival of jobs in the morning is unnecessary because jobs will still be available in the evening. By contrast, if a worker believes that others are waking up early to monitor new postings, then doing the same becomes necessary to avoid falling to the back of the queue. Increasing returns to early search enables multiplicity. That multiplicity is more likely in a tight labor market follows from the fact that, regardless of when others search, the value of doing the same (i.e., playing the equilibrium strategy) is greater when there is greater competition for workers (i.e., high θ), whereas the value of searching before or after everyone else (i.e., deviating from equilibrium play) is not directly affected by the degree of competition for workers—monitoring in the morning when others wait until the evening *guarantees* finding a job whereas waiting until the evening when others search in the morning *precludes* finding a job. It follows that when competition for workers is high, deviating from equilibrium play is less attractive, thus sustaining multiplicity.

The second main result of the paper is that the multiplicity introduced by the availability of monitoring technologies can destabilize the labor market. To demonstrate this point, I embed a version of the monitoring game described above in a quantitative equilibrium model of the labor market. As in the game above, searchers actively monitor the arrival of vacancies—the result of which is a frictionless matching process for workers who monitor—while other unemployed workers only have access to a standard frictional matching process operating at the end of each period. Monitoring technologies in the quantitative model thus function to (i) enable workers who monitor to match quickly (as highlighted in the model and discussion above) and (ii) reduce matching frictions. The model also features dynamic processes for unemployment and vacancies, endogenous determination of wages via Nash bargaining, and an entrepreneurial sector giving rise to a flexible model of vacancy creation that nests various entry processes considered in the literature as special cases (e.g., free entry and "Diamond entry"). When no workers monitor the arrival of new vacancies, the model reduces to a standard search and matching model of the labor market in the Diamond-Mortensen-Pissarides (DMP) tradition,[3] augmented with a generalized vacancy creation process. The model is thus well-suited to study the role of monitoring in a general equilibrium environment, whether multiplicity continues to arise, and if so, the implications for labor market dynamics.

There are two important features of the quantitative model that will tend to weaken the source of multiplicity that pervades the monitoring game in Table 1: Endogenous wages and entry of firms.[4] In the quantitative model, as more workers choose to monitor the arrival of vacancies, wages fall. This happens because, if workers believe other workers are monitoring, they expect to have to do the same if they become unemployed—and thus to have to forgo leisure—in order to avoid falling to the back of the queue. This weakens their bargaining position and depresses wages. Depressed wages, in turn, stimulate job creation, which alleviates congestion among those at the back of the queue for jobs, thus reducing the urgency of monitoring and mitigating the increasing returns that initially gave rise to multiplicity. Thus, if job creation is sufficiently responsive to wages, multiplicity will cease to exist. Indeed, in the limiting case of free entry (in which the elasticity of job creation is infinite), there is a unique equilibrium. By contrast, when the generalized job creation process that I consider is calibrated to match estimates of the elasticity of job creation in empirical studies, the calibrated model has three steady-state equilibria.

Across the three steady states, wages and unemployment are positively correlated. This observation reflects differences in labor supply sustained by self-confirming beliefs among unemployed workers. Specifically, each steady state is associated with different beliefs among workers about how dire their job-finding prospects will be if they do not actively monitor the arrival of vacancies and are thus relegated to the back of the queue for jobs. These different beliefs induce different monitoring decisions that, in equilibrium, confirm the beliefs that led to them, thus sustaining different levels of labor supply across steady states. Moreover, the differences in beliefs across steady states, and the associated monitoring decisions, have normative consequences: In wide regions of the parameter space, social welfare is *negatively* correlated with monitoring (and thus positively correlated with unemployment) across steady states, reflecting strong negative congestion externalities from monitoring that need not be fully offset by the matching efficiency gains that monitoring technology yields. That this can occur is noteworthy because it reverses the Pareto-ranking in the canonical treatment of search-induced multiplicity in Diamond (1982a), where high-activity steady states necessarily Pareto dominate low-activity ones.

How do monitoring technologies potentially destabilize the labor market? In addition to having multiple steady state equilibria, the model also implies multiple *dynamic* equilibria, in the sense that, from a range of initial states, the economy can converge to different steady states. This can function to destabilize the economy in two ways. First, it implies that non-fundamental changes in beliefs among workers can permanently change the path of the economy, from a trajectory converging to one steady state to a trajectory converging to another. In this way, the model provides a novel theory of endogenous labor supply shocks. Second, the quantitative model also implies that when the unemployment rate is sufficiently high, there is only one equilibrium path—the path leading to the high-monitoring, low-unemployment steady state. This, in turn, implies

[3]Diamond (1982b), Mortensen (1982), and Pissarides (1985).

[4]Recall that, in the game in Table 1, both θ and w are treated as parameters, whereas both are endogenized in the quantitative model.

that a sufficiently large adverse demand shock can force the economy onto a path associated with a permanently higher level of monitoring than the path it began on. Put more simply, large demand shocks can permanently affect labor supply.

This second observation can potentially offer insight into the recovery from the Great Recession. Specifically, in October of 2009, the unemployment rate rose to a 25-year high of 10%. The subsequent recovery ultimately resulted in unemployment falling substantially *below* its pre-recession level and the vacancy rate rising substantially *above* its pre-recession level. Meanwhile, as was noted at the time,[5] even as late as 2019, wage growth remained weak relative to what would have been expected based on vacancy and unemployment data. These observations of tepid wage growth despite high vacancies and low unemployment are difficult to square with both traditional models of the labor market and models of multiple steady states generated by demand-side mechanisms.[6] The mechanism that I study—whereby the availability of monitoring technologies changes the nature of search and exposes the economy to multiplicity—provides a simple alternative explanation. In the quantitative model, a demand shock that drives the economy to 10% unemployment *necessarily* forces the economy to the high-monitoring steady state. This implies not only a slow recovery, but also one that overshoots the original levels of unemployment and vacancies from which the economy started, and in which wages never fully recover.

By its nature, monitoring is difficult to directly identify in the data. Doing so would require real-time data matching the timing of vacancy postings to the timing of workers' applications.[7] Nevertheless, I document two features of the data that are suggestive of monitoring and its implications. First, I use high-frequency panel data on unemployed workers' search decisions to document that online job search is significantly less lumpy than traditional job search at a daily frequency. That is, workers using the internet for search are significantly more likely to search on any given day than others, even after controlling for time spent on job search and job-seeker fixed effects. This is consistent with online job search allowing workers to monitor the arrival of new job openings and thus apply to new jobs *as they arrive*, rather than economizing on fixed costs and, e.g., devoting a single day of the week to job search. Second, I revisit the macroeconomic SVAR literature seeking to identify labor supply shocks in aggregate data. Using a standard sign restriction approach to identification, I document that the contribution of labor supply shocks to aggregate fluctuations at all horizons increased markedly around the turn of the century. This is consistent with the emergence of endogenous belief-driven fluctuations in job search driven by the rapid growth in online search beginning in the early 2000s. While these pieces of evidence are by no means conclusive, they are, respectively, suggestive of monitoring and its implications.

The paper proceeds as follows: Section 2 studies a stylized monitoring game among unemployed workers and illustrates how monitoring leads to multiplicity. Section 3 embeds the game in a quantitative model of the labor market. Section 4 calibrates the model and considers its steady states. Section 5 turns to local and global dynamics, focusing on how endogenous belief-driven fluctuations can emerge and the model's implications for the recovery from the Great Recession.

[5] For example, see John Robertson's discussion in a post on the Atlanta Fed's Macroblog on November 25, 2019.

[6] Traditional models in which the economy converges to a unique steady state will struggle to explain this "overshooting" absent permanent exogenous shocks. Models with multiplicity of steady states associated with demand-side mechanisms will tend to imply a *negative* correlation between unemployment and wages across steady states.

[7] Davis and Samaniego de la Parra (2020) have such data and find strong evidence of "application bunching," whereby new vacancies receive a disproportionate number of applications immediately after being posted. This is precisely what the monitoring technologies studied in this paper would predict.

Section 6 documents two novel pieces of evidence consistent with the model and its implications.

2 Monitoring Game

The central insight of this paper is that monitoring technologies can lead to increasing returns in the matching function and thus multiple equilibria. To illustrate the mechanism, I begin by considering a stylized game between unemployed workers with access to a monitoring technology. I first describe the environment, then characterize equilibria of the game, and finally discuss the implications for multiplicity.

2.1 Environment

Consider a one-period game among u unemployed workers competing to match with $v < u$ vacancies that are posted sequentially at random times throughout the period. Workers who are matched by the end of the period receive payoff w and workers who are unmatched receive payoff $b < w$.

2.1.1 Monitoring decision

At the start of the period, each worker i draws flow utility of leisure κ_i from distribution F with support $[\underline{\kappa}, \bar{\kappa}]$. After observing κ_i, workers decide how to spend their time: They can either actively monitor the arrival of vacancies throughout the period—thus gaining a first-mover advantage in submitting applications—or enjoy leisure and wait until the end of the period to try to find work. Workers choosing to monitor match with probability p^m but forgo leisure, while workers choosing to wait match with probability $p^w < p^m$ but enjoy flow utility from leisure κ_i. The values of monitoring and waiting for worker i, respectively, are thus

$$U^m \qquad = p^m w + (1 - p^m)b \tag{1}$$

$$U^w(\kappa_i) = p^w w + (1 - p^w)b + \kappa_i. \tag{2}$$

Workers monitor if and only if the value of monitoring exceeds the value of waiting: $U^m > U^w(\kappa_i)$. Equations (1) and (2) reveal that monitoring decisions are characterized by a cutoff rule such that a worker drawing $\kappa_i < \kappa_i^*$ will choose to monitor a worker drawing $\kappa_i \geq \kappa_i^*$ will choose to wait, where (when interior) κ_i^* is defined by $\kappa_i^* \equiv \{\kappa_i | U^m = U^w(\kappa_i)\}$. From (1) and (2), we have

$$\kappa_i^* = \begin{cases} \underline{\kappa} & \text{if } \big(p^m - p^w\big)(w - b) < \underline{\kappa} \\ \big(p^m - p^w\big)(w - b) & \text{if } \big(p^m - p^w\big)(w - b) \in [\underline{\kappa}, \bar{\kappa}] \\ \bar{\kappa} & \text{if } \big(p^m - p^w\big)(w - b) > \bar{\kappa} \end{cases} \tag{3}$$

where, implicitly, the match rates p^m and p^w depend on the equilibrium behavior of other workers in the economy. I next derive expressions for these two match rates.

2.1.2 Matching

The preceding implies that matching occurs in two phases: A monitoring phase and an aftermarket. Because vacancies arrive sequentially throughout the period and monitoring allows workers to immediately observe and apply to newly posted jobs, matching in the monitoring phase is frictionless. Letting κ^* denote the (symmetric) equilibrium monitoring cutoff, $F(\kappa^*)u$ is the number of monitoring workers and the number of matches in the monitoring phase is

$$\mathring{m} = \min\left\{v, F(\kappa^*)u\right\} \tag{4}$$

implying that the match rate for workers in the monitoring phase is $\mathring{p} \equiv \frac{\mathring{m}}{F(\kappa^*)u}$.

For simplicity, I assume that the aftermarket is likewise frictionless.[8] Thus, the number of matches in the aftermarket is given by

$$\hat{m} = v - \mathring{m} \tag{5}$$

implying that the match rate for workers in the aftermarket is $\hat{p} \equiv \frac{\hat{m}}{u - F(\kappa^*)u}$.

Workers who choose to monitor are able to match during the monitoring phase and, failing that, in the aftermarket[9] Workers who choose to wait are only able to match in the aftermarket. This implies that the match rates for monitoring workers and waiting workers that appear in (3) are, respectively, given by $p^m = \mathring{p} + (1 - \mathring{p})\hat{p}$ and $p^w = \hat{p}$. Using these together with (4) and (5), and defining market tightness as $\theta \equiv v/u$, it is straightforward to show that the match rates for monitoring workers and waiting workers, respectively, can be written as

$$p^m = \begin{cases} 1 & \text{if} \quad F(\kappa^*) < \theta \\ \frac{\theta}{F(\kappa^*)} & \text{if} \quad F(\kappa^*) \geq \theta \end{cases} \tag{6}$$

$$p^w = \begin{cases} \frac{\theta - F(\kappa^*)}{1 - F(\kappa^*)} & \text{if} \quad F(\kappa^*) < \theta \\ 0 & \text{if} \quad F(\kappa^*) \geq \theta. \end{cases} \tag{7}$$

The model is fully described by equations (3), (6), and (7).

2.1.3 Increasing returns

Before characterizing the Nash equilibria of this game, it is useful to reflect on how the matching process gives rise to increasing returns. Observe that the match rates in (6) and (7) imply that the return to monitoring is proportional to

$$p^m - p^w = \begin{cases} \frac{1-\theta}{1-F(\kappa^*)} & \text{if} \quad F(\kappa^*) < \theta \\ \frac{\theta}{F(\kappa^*)} & \text{if} \quad F(\kappa^*) \geq \theta. \end{cases} \tag{8}$$

Equation (8) reveals that, when the labor market is tight ($\theta > F(\kappa^*)$), the model exhibits increasing returns in the sense that the individual benefit from monitoring relative to waiting, $p^m - p^w$, is an increasing function of the equilibrium monitoring cutoff κ^*. As has been well known since the seminal work of Diamond (1982a), this property can expose the economy to multiple equilibria.

Monitoring technologies can lead to increasing returns because of the first-mover advantage that they offer searchers. In a tight labor market ($\theta > F(\kappa^*)$), the existence of a first-mover advantage *reduces* congestion among workers who monitor ($p^m = 1$) at the expense of *increased* congestion among those who wait, all of whom are forced to compete for a diminished stock of remaining jobs. As a consequence, as more workers monitor, the job-finding prospects of workers waiting until the

[8]I relax this assumption in the quantitative model in Section 3.

[9]This assumption is inessential.

end of the period deteriorate increasingly rapidly ($\frac{\partial p^w}{\partial \kappa^*} < 0$, $\frac{\partial^2 p^w}{\partial \kappa^{*2}} < 0$), thus causing the return to monitoring to rise increasingly rapidly ($\frac{\partial p^m - p^w}{\partial \kappa^*} > 0$, $\frac{\partial^2 p^m - p^w}{\partial \kappa^{*2}} > 0$). This implies increasing returns.

2.2 Nash equilibria

From (6) and (7), we see that the match rates p^m and p^w depend on the cutoff chosen by other workers in the economy. Thus, with a slight abuse of notation, $p^m = p^m(\kappa^*)$ and $p^w = p^w(\kappa^*)$. Accordingly, we can write the optimal cutoff of worker i in equation (3) as $\kappa_i^* = \kappa_i^*(\kappa^*)$ and interpret this as the best-response function of worker i. The (symmetric) Nash equilibria of the game are then the fixed points of this best-response function, i.e. the points at which $\kappa_i^*(\kappa^*) = \kappa^*$.

To characterize the set of equilibria, substitute (6) and (7) into (3) to obtain the best-response function expressed explicitly as a function of κ^*:

$$\kappa_i^*(\kappa^*) = \begin{cases} \kappa^L(\kappa^*) & \text{if} \quad F(\kappa^*) \leq \theta \\ \kappa^U(\kappa^*) & \text{if} \quad F(\kappa^*) > \theta \end{cases} \tag{9}$$

where

$$\kappa^L(\kappa^*) \equiv \begin{cases} \underline{\kappa} & \text{if} \quad \frac{1-\theta}{1-F(\kappa^*)}(w-b) < \underline{\kappa} \\ \frac{1-\theta}{1-F(\kappa^*)}(w-b) & \text{if} \quad \frac{1-\theta}{1-F(\kappa^*)}(w-b) \in [\underline{\kappa}, \bar{\kappa}) \end{cases} \tag{10}$$

$$\kappa^U(\kappa^*) \equiv \begin{cases} \frac{\theta}{F(\kappa^*)}(w-b) & \text{if} \quad \frac{\theta}{F(\kappa^*)}(w-b) \in (\underline{\kappa}, \bar{\kappa}] \\ \bar{\kappa} & \text{if} \quad \frac{\theta}{F(\kappa^*)}(w-b) > \bar{\kappa}. \end{cases} \tag{11}$$

Equations (9), (10) and (11) indicate that $\kappa_i^*(\kappa^*)$ is continuous, has a kink at $\kappa^c \equiv F^{-1}(\theta)$, is weakly increasing for $\kappa^* \leq \kappa^c$, and is weakly decreasing for $\kappa^* > \kappa^c$. In general, the number of equilibria will depend on the shape of the distribution function F. Nevertheless, Proposition 1 provides a necessary condition and a sufficient condition for multiplicity.

PROPOSITION 1 (Necessary and sufficient conditions for multiplicity). *Assume that (i) F is strictly increasing and (ii) $\bar{\kappa} < w - b$ (i.e., the type who derives the greatest utility from leisure will monitor if guaranteed to find a job). Then:*

1. *There is a value $\bar{\theta} < 1$ such that for any $\theta > \bar{\theta}$ there are multiple equilibria.*
2. *There is a value $\underline{\theta} > 0$ such that for all $\theta < \underline{\theta}$ an equilibrium exists, it is unique, and it satisfies $\kappa^* > \kappa^c$. That is, the unique equilibrium entails high monitoring.*

Proof. See Appendix A. □

Proposition 1 establishes two important features of monitoring: First, monitoring can generate multiple equilibria. This is consistent with the discussion of increasing returns above. Second, the tendency for multiple equilibria to emerge is intimately related to market tightness. Precisely, multiplicity is guaranteed by a sufficiently tight labor market and precluded by a sufficiently slack one. Both parts of this result will have quantitative analogs with important macroconomic implications in the fully dynamic model that I develop in Section 3.

2.3 Discussion

Both parts of Proposition 1 follow from a general feature of monitoring: Monitoring temporally segments markets (as described in Section 2.1), which implies that deviations from equilibrium play entail payoffs that are disconnected from market forces. This implies that a tight labor market encourages equilibrium play while a slack one discourages it.

Consider a worker deciding between playing the equilibrium strategy and deviating when all other workers monitor. In this case, market tightness is only payoff-relevant when she plays the equilibrium strategy (monitoring), since deviating (waiting) precludes finding a job and is thus independent of market conditions. Likewise, for a worker deciding between playing the equilibrium strategy and deviating when all other workers wait, market tightness is only payoff-relevant when she plays the equilibrium strategy (waiting), since deviating (monitoring) guarantees that she will find a job and is thus independent of market conditions. In both cases, favorable market conditions—high θ—increase the value of playing the equilibrium strategy.

2.4 Traditional model

The traditional approach to modeling job search decisions in macroeconomic models is to assume that searchers and non-searchers are perfect substitutes in the match function. This implies that non-searchers are simply ineffective or low-effort searchers, matching at a rate proportional to that of searchers.[10] Consider such a model in the context of an environment similar to the one described in Section 2.1 but with a more standard matching process—replacing p^m with p^h ("high" effort) and p^w with p^l ("low" effort). Letting $z < 1$ denote effort of non-searchers relative to searchers, then, continuing to denote by $F(\kappa^*)$ the fraction of workers choosing to search in equilibrium, the effective measure of searchers in the economy is

$$\begin{aligned} s(\kappa^*) &= F(\kappa^*)u + (1 - F(\kappa^*))zu && (12) \\ &= \tilde{s}(\kappa^*)u && (13) \end{aligned}$$

where $\tilde{s}(\kappa^*) \equiv F(\kappa^*) + (1 - F(\kappa^*))z$ is average effective search effort. Maintaining the assumption of frictionless matching, the number of matches is $m = \min\{v, \tilde{s}(\kappa^*)u\}$, which implies match rates

$$p^h = \begin{cases} 1 & \text{if} \quad \tilde{s}(\kappa^*)u < v \\ \frac{v}{\tilde{s}(\kappa^*)u} & \text{if} \quad \tilde{s}(\kappa^*)u \geq v \end{cases} \tag{14}$$

$$p^l = \begin{cases} z & \text{if} \quad \tilde{s}(\kappa^*)u < v \\ z\frac{v}{\tilde{s}(\kappa^*)u} & \text{if} \quad \tilde{s}(\kappa^*)u \geq v. \end{cases} \tag{15}$$

Inspection of (14) and (15) in light of (12) and $z < 1$ reveals that the return to search, $p^h - p^l$, is a decreasing function of the number of searchers, κ^*, which implies that there cannot be multiplicity.

PROPOSITION 2 (No multiplicity). *There is at most one equilibrium in the traditional model.*

Proof. See Appendix A. □

Thus, monitoring is the essential feature of the environment that gives rise to multiple equilibria. I next turn to a fully dynamic model to study the quantitative implications of monitoring technology.

[10] See, for example, Krusell et al. (2017) and Cairo et al. (2021).

3 Dynamic Model

In this section, I embed a variation on the static game described in Section 2 within a dynamic equilibrium model of the labor market.

3.1 Environment

Time is discrete and runs forever. The economy is populated by a unit measure of ex ante identical workers who are either employed or unemployed and a fixed measure of entrepreneurs who create vacancies. Workers and entrepreneurs all discount the future with discount factor β, have perfect foresight with respect to the evolution of aggregate variables, and seek to maximize the present discounted value of lifetime utility.

Important elements of the model below—in particular, those relating to workers' decisions and the matching process—are largely unchanged from the simple game described previously in Section 2. In order for the analysis in this section to be self-contained, I provide a full description of these elements of the model below.

3.1.1 Accounting

Let u_t denote the total number of unemployed workers in a period and v_t the total number of vacancies in a period. Unemployed workers either actively monitor the arrival of vacancies throughout the period (u_t^m) or wait until the end of the period to match (u_t^w). Vacancies are either newly posted by an entrepreneur (v_t^n) or were posted in a previous period and are therefore old (v_t^o). Thus, we have the following accounting identities:

$$u_t = u_t^m + u_t^w \tag{16}$$

$$v_t = v_t^n + v_t^o. \tag{17}$$

Unemployed workers and vacancies are matched through a frictional matching process. Matches become productive in the period after they are formed and are destroyed with probability δ at the end of each period (including matches that have formed but have not yet become productive). Letting m_t denote the total number of matches in a period, unemployment and vacancies thus evolve according to the following laws of motion:

$$u_{t+1} = u_t + \delta(1 - u_t) - (1 - \delta)m_t \tag{18}$$

$$v_{t+1} = (1 - \delta)(v_t - m_t) + v_{t+1}^n. \tag{19}$$

Equations (18) and (19) are standard and describe the aggregate dynamics of unemployment and vacancies in the model.

3.1.2 Matching

As in Section 2, unemployed workers choosing to monitor new job postings are able to observe (and apply to) new jobs as soon as they become available—and thus before other unemployed workers who are *not* actively monitoring new postings. Monitoring thus results in a temporally segmented matching process that can be thought of as taking place in two phases: A monitoring phase (which lasts for the duration of the period) followed by an aftermarket phase (at the end of the period).

Monitoring. Newly created vacancies arrive randomly throughout each period. During the monitoring phase, as newly posted vacancies arrive to the market, they are matched with workers who have chosen to monitor those arrivals. Because vacancies arrive sequentially throughout the period and monitoring allows workers to see new vacancies immediately, the matching process in the monitoring phase is frictionless and the total number of matches is given by[11]

$$\mathring{m}_t = \min\{v_t^n, u_t^m\}. \tag{20}$$

The corresponding match rates for workers and vacancies in the monitoring phase are

$$\mathring{p}_t = \mathring{m}_t / u_t^m \tag{21}$$

$$\mathring{q}_t = \mathring{m}_t / v_t^n. \tag{22}$$

Aftermarket. At the end of each period—after all new vacancies have been posted—the aftermarket opens. In the aftermarket, all remaining unmatched workers (monitoring workers who failed to match while monitoring and workers who did not monitor) match with all unmatched vacancies (newly posted vacancies that failed to match with monitoring workers and old vacancies that were posted in previous periods and thus were not observed by monitoring workers). Thus, the total number of matches in the aftermarket is given by[12]

$$\hat{m}_t = \mu\big(v_t - \mathring{m}_t, u_t - \mathring{m}_t\big). \tag{23}$$

where μ is a matching function. The corresponding match rates for workers and vacancies in the aftermarket are

$$\hat{p}_t = \hat{m}_t / (u_t - \mathring{m}_t) \tag{24}$$

$$\hat{q}_t = \hat{m}_t / (v_t - \mathring{m}_t). \tag{25}$$

The preceding implies that the total number of matches in the economy is given by

$$m_t = \hat{m}_t + \mathring{m}_t \tag{26}$$

with corresponding average match rates $p_t = m_t/u_t$ and $q_t = m_t/v_t$. Notice that the matching process described above reduces to a standard matching model in the absence of monitoring technology: That is, if $\mathring{m}_t = 0$, then the total number of matches is just $m_t = \mu(v_t, u_t)$. Thus, the model is simply a generalization of a traditional matching model with no search decision, in which monitoring technology affords searchers a first-mover advantage. I next consider workers' optimal choice to use this monitoring technology.

3.1.3 Workers

Employment and unemployment. Workers are either employed or unemployed. Employed workers receive wage w_t at the beginning of each period. Unemployed workers receive income $b_t = bw_t$ at the beginning of each period and decide whether or not to monitor new job postings.

[11] I denote variables corresponding to the monitoring phase with circles, e.g. $\mathring{x}$.

[12] I denote variables corresponding to the aftermarket with hats, e.g. $\hat{x}$.

Workers who do not monitor new postings draw i.i.d. flow utility κ_{it} (e.g., leisure or home production) from distribution F and are only able to match in the aftermarket at the end of the period. Workers who monitor new postings, on the other hand, do not get to draw flow utility κ_{it}, but have the opportunity to match during the monitoring phase with newly posted vacancies in addition to being able to match in the aftermarket if they fail to match while monitoring. Thus, once again letting p_t^w denote the match rate for workers who wait until the aftermarket rather than monitoring and p_t^m the match rate for workers who monitor, we have:

$$p_t^m = \mathring{p}_t + (1 - \mathring{p}_t)\hat{p}_t \tag{27}$$

$$p_t^w = \hat{p}_t. \tag{28}$$

Monitoring decision. Let W_t denote the value of entering period t employed, U_{it} the value of entering period t unemployed, U_t^m the value of being unemployed and monitoring new postings throughout the period, and U_{it}^w the value of being unemployed and waiting until the aftermarket to match. Then the description of the environment above implies:

$$W_t = w_t + \Big[\delta\beta\mathbb{E}U_{it+1} + (1-\delta)\beta W_{t+1}\Big] \tag{29}$$

$$U_{it} = b_t + \max\Big\{U_t^m, U_{it}^w\Big\} \tag{30}$$

$$U_t^m = p_t^m\Big[\delta\beta\mathbb{E}U_{it+1} + (1-\delta)\beta W_{t+1}\Big] + (1-p_t^m)\Big[\beta\mathbb{E}U_{it+1}\Big] \tag{31}$$

$$U_{it}^w = p_t^w\Big[\delta\mathbb{E}\beta U_{it+1} + (1-\delta)\beta W_{t+1}\Big] + (1-p_t^w)\Big[\beta\mathbb{E}U_{it+1}\Big] + \kappa_{it}. \tag{32}$$

where the expectation operator reflects uncertainty regarding future (i.i.d.) draws of κ_{it}. As we see in (31) and (32), an unemployed worker's decision about whether or not to monitor the arrival of new postings entails a tradeoff between a higher probability of matching if she chooses to monitor ($p_t^m \geq p_t^w$), and higher flow utility if she cannot to monitor ($b_t + \kappa_{it} \geq b_t$).

Unemployed workers choose to monitor the arrival of new postings if and only if the value of doing so exceeds the value of not doing so, i.e. $U_t^m > U_{it}^w$. Because κ_{it} is i.i.d., (31) and (32) imply that the monitoring decision takes the form of a cutoff rule such that workers choose to monitor if and only if the value of leisure exceeds a threshold defined by the value of κ_{it} such that $U_t^m = U_{it}^w$:

$$\kappa_{it}^* = \beta(1-\delta)(p_t^m - p_t^w)\mathbb{E}\Big[W_{t+1} - U_{it+1}\Big]. \tag{33}$$

In a symmetric equilibrium $\kappa_{it}^* = \kappa_t^*$ for all i. Thus, the measure of monitoring and waiting workers in period t are given, respectively, by

$$u_t^m = F(\kappa_t^*)u_t \tag{34}$$

$$u_t^w = (1 - F(\kappa_t^*))u_t. \tag{35}$$

3.1.4 Entrepreneurs and job creation

I consider a generalized job creation process that parameterizes the elasticity of job creation and thus nests free-entry and inelastic job creation processes as limiting cases.[13] Specifically, there is a fixed measure η of entrepreneurs in the economy, each of whom can invest in the creation of a new job (i.e., create a vacancy). During each period t, each such entrepreneur j receives an opportunity to draw i.i.d. sunk investment cost ξ_{jt} from distribution G. If an entrepreneur decides to invest, it pays the sunk cost and creates a new vacancy with value V_t^n. Entrepreneurs undertake the investment if the expected value of creating a new vacancy exceeds the sunk investment cost, i.e., if $V_t^n > \xi_{jt}$. Thus, the measure of newly created vacancies in the economy, v_t^n, is determined by

$$v_t^n = \eta G\big(V_t^n\big) \tag{36}$$

where, if we let q_t^n denote the match rate for new vacancies and q_t^o denote the match rate for old vacancies, the value of a new vacancy, an old vacancy, and a filled job, respectively, are given by

$$V_t^n = -c + \beta(1-\delta)\Big[q_t^n J_{t+1} + (1-q_t^n)V_{t+1}^o\Big] \tag{37}$$

$$V_t^o = -c + \beta(1-\delta)\Big[q_t^o J_{t+1} + (1-q_t^o)V_{t+1}^o\Big] \tag{38}$$

$$J_t = y_t - w_t - \tau_t + \beta(1-\delta)\Big[J_{t+1}\Big]. \tag{39}$$

where τ_t is a lump-sum tax used to finance UI payments b_t to unemployed workers. Note that the description of the matching process above implies that q_t^n and q_t^o are given by

$$q_t^n = \mathring{q}_t + (1-\mathring{q}_t)\hat{q}_t \tag{40}$$

$$q_t^o = \hat{q}_t. \tag{41}$$

Considering a generalized job creation process described in (36) is important for the existence of multiple equilibria. In particular, in the free-entry limit, vacancy creation is highly responsive to workers' monitoring decisions, and because the incentives to monitor diminishes with the number of firms in the economy (all else equal), a highly elastic job creation process precludes multiplicity. As I describe in detail below, I calibrate the elasticity of the entry process to match empirical estimates of the wage elasticity of labor demand.

3.1.5 Wages

Wages are determined by Nash bargaining. Letting χ denote workers' bargaining power, the Nash bargained wage solves

$$(1-\chi)\Big[W_t - \mathbb{E}U_{it}\Big] = \chi\Big[J_t - V_t^o\Big]. \tag{42}$$

Equation 42 reflects the fact that, if bargaining breaks down, a vacancy is no longer new and thus the outside option for any firm is V_t^o. This implies that the wage is the same in all matches.

[13]See Fujita and Ramey (2005), Beaudry et al. (2018) and Coles and Moghaddasi Kelishomi (2018) for examples of similar processes.

3.1.6 Taxes and transfers

The government is assumed to run a balanced budget, so that total transfers to unemployed workers are paid for by lump-sum tax on firms τ_t:

$$\tau_t = \frac{b w_t u_t}{1 - u_t}. \tag{43}$$

3.2 Equilibrium

We are now prepared to formally define a perfect foresight equilibrium of the dynamic model.

Definition 1. *A (symmetric) perfect foresight equilibrium is a sequence* $\{u_t, v_t, \kappa_t^*, v_t^n\}$ *satisfying* (18),(19), (33), *and* (36) *for all* $t \geq 1$, *given:*

1. *The definitions and equilibrium conditions in* (16)-(43)*;*
2. *Initial conditions:* $\{u_0, v_0\}$*;*
3. *Transversality conditions:* $\lim_{t\to\infty} v_t^n < \infty$ *and* $\lim_{t\to\infty} \kappa_t^* < \infty$.

4 Calibration and Steady States

With the full dynamic model in hand, I turn to calibrating the model's structural parameters and studying the model's steady-state equilibria.

4.1 Calibration

Below I describe the functional forms and baseline parameter values used to assess the model's quantitative implications.

4.1.1 Functional forms

I assume that the distribution of the flow value of non-work, F, is Log-Normal with parameters μ_κ and σ_κ, both of which I calibrate to match moments in the data below. Following Beaudry et al. (2018) and Coles and Moghaddasi Kelishomi (2018), I adopt a flexible form for the entrepreneurial investment cost function,

$$G(\xi_{jt}) = \left(\xi_{jt}/\bar{\xi}\right)^{\nu}. \tag{44}$$

The parameter ν governs the elasticity of job creation, allowing (44) to nest both free-entry and an inelastic vacancy creation process as special cases.[14] Finally, I assume that workers and vacancies that remain unmatched at the end of each period are matched via an urn-ball matching function,

$$\mu\big(v_t - \mathring{m}_t, u_t - \mathring{m}_t\big) = (v_t - \mathring{m}_t)\left[1 - e^{-\psi\frac{u_t - \mathring{m}_t}{v_t - \mathring{m}_t}}\right]. \tag{45}$$

This functional form has the virtue of remaining bounded between 0 and 1, which is particularly important when there are potentially multiple equilibria, and furthermore naturally nests the fully

[14] The parameter $\bar{\xi} \equiv \max\{\xi_{jt}\}$ ensures that this function can be interpreted as a proper distribution function. However, from (36) we see that $\mathring{v}_t = \eta G(\mathring{J}_t^u) = \eta(\mathring{J}_t^u/\bar{\xi})^{\nu} = \eta\bar{\xi}^{-\nu}(\mathring{J}_t^u)^{\nu}$, implying that this parameter is not separately identified from η. Thus, in Table 2, I report the calibrated value of $\tilde{\eta} \equiv \eta\bar{\xi}^{-\nu}$ rather than η.

frictionless matching process described in Section 2 as the matching efficiency parameter ψ becomes large.

4.1.2 Parameter values

I calibrate the model's parameters to values commonly used in the literature and moments in the data. The task of calibrating the model is made somewhat more difficult than usual because of two unique features of the environment on which I focus: (i) Because the model is intended to describe an economy in which monitoring technologies are available to unemployed workers, care must be taken to use relatively recent data for calibration, and (ii) because of the possibility of multiple steady states, to the extent that macro data is used for calibration, it will be necessary to take a stance on which steady state of the model is assumed to be generating the data. Both issues are alleviated by directly calibrating structural parameters to match, e.g., estimates from micro data—I do this whenever possible. For the remaining parameters that I calibrate indirectly, I restrict attention to relatively recent data/estimates or studies that otherwise control for time fixed effects, and assume that the economy is in the low-monitoring steady state (if multiple steady states exist). Table 2 reports these values. See Appendix C for details of the calibration procedure.

Table 2: Calibration

Concept	Parameter	Value
Discount factor	β	0.997
Separation rate	δ	0.02
UI replacement rate	b	0.36
Leisure/home production (mean)	μ_κ	-0.87
Leisure/home production (std. dev.)	σ_κ	0.13
Productivity	y	1
Bargaining parameter (labor's share)	χ	0.7
Vacancy posting cost	c	0.17
Job creation (scale)	$\tilde{\eta} \equiv \eta\bar{\xi}^{-\nu}$	0.02
Job creation (elasticity)	ν	0.03
Matching efficiency	ψ	1.11

Notes: Monthly frequency. Moments computed by the author for calibration based on data between 2000 and 2007.

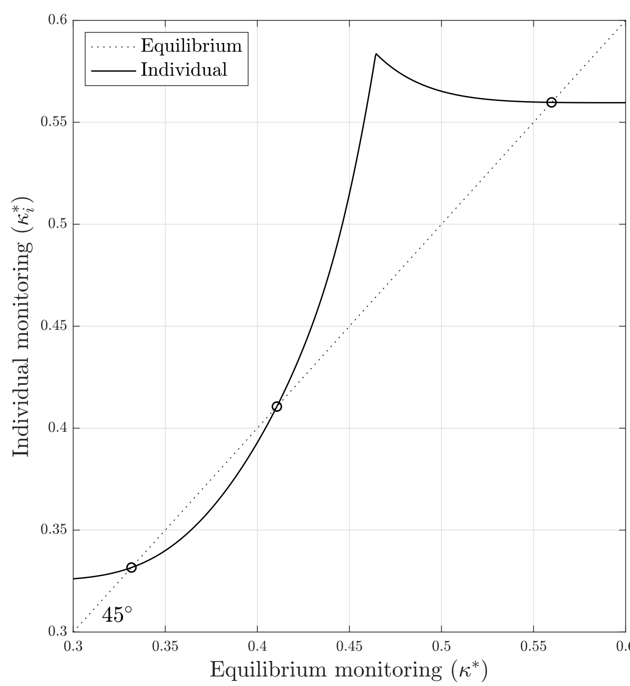
Labor productivity is normalized to $y = 1$. I choose a discount factor of $\beta = 0.997$ consistent with a 4% annual steady state interest rate. I use a value of $b = 0.36$ following Anderson and Meyer (1997) who compute a replacement rate of 36% based on statutory provisions of the UI system. I set the vacancy posting cost to $c = 0.17$, the value computed by Fujita and Ramey (2012) based on data from Barron et al. (1997). I choose a value for workers' bargaining power of $\chi = 0.7$ following, e.g., Shimer (2005)

I calibrate the remaining parameters to match a set of moments from the data and other studies. I choose the first two parameters to match moments computed using averages of data from 2000 until 2007:[15] I choose the match efficiency parameter, ψ, to match the average monthly job-finding probability of 0.36,[16] and the monthly separation rate, δ, to match the average unemployment rate of 5.3%. I set the mean of the (log) flow value of non-work in the model, μ_κ, to yield an average flow value of non-work of 75% of labor productivity, based on the midpoint of the range identified by Chodorow-Reich and Karabarbounis (2016) and similar to the preferred value of Hall and Milgrom (2008). I choose the standard deviation of the (log) flow value of non-work, σ_κ, to match data from Davis and Samaniego de la Parra (2020), who find that roughly 10% of vacancy postings last for under 24 hours.[17] I choose the parameter scaling the entry process, $\tilde{\eta}$, to match the average *posting* duration, also in Davis and Samaniego de la Parra (2020), of 9 days.[18] Finally, I choose the elasticity parameter of the job creation process ν to match various estimates of the wage elasticity of labor demand from the literature.

4.2 Steady-state equilibria

The steady-state equilibria of the full dynamic model cannot be solved for analytically, so I proceed numerically. Figure 1 plots the (steady-state) best response function for worker i based on equation (33). As usual, the intersection of the best-response function an the 45-degree line identifies the

Figure 1: Best response function



equilibria of the monitoring game. In this case, there are three intersections, corresponding to the

[15] I choose these dates because they precede the Great Recession and allow the use of JOLTS data.

[16] I compute the monthly hazard rate using data on (i) the total number of unemployed workers (u_t) and (ii) the number of short-term unemployed workers (u_t^{ST}) via $p_t = 1 - \frac{u_{t+1} - u_{t+1}^{ST}}{u_t}$, following the method of Shimer (2012).

[17] See van Ours and Ridder (1992), Faberman and Menzio (2018), and Mueller et al. (2020) for evidence on negative duration dependence in vacancy exit hazards.

[18] See Footnote 14 for a discussion of why I calibrate $\tilde{\eta}$ rather than η.

three steady-state values of κ^*.

What are the properties of the three steady states observed in Figure 1? Table 3 reports the implied values of the targeted moments for all three steady states: The unemployment rate (u), the average job-finding probability (p), labor market tightness (θ). It also reports the implied values of several other moments of interest: The share monitoring ($F(\kappa^*)$), the wage (w), job creation (v^n/v),[19] the job-finding probability for monitoring workers (p^m), and the job-finding probability for non-monitoring workers (p^w).

Table 3: Description of equilibria

	Data	Steady-state		
		Low-κ^*	*Int.-*κ^*	*High-*κ^*
Targeted moments				
Unemployment rate (u)	5.3%	5.3%	3.4%	2.2%
Job-finding prob. (p)	0.36	0.36	0.58	0.91
Average posting duration	9	9	4.9	0.0
% imm. matched firms	10%	10%	75%	100%
Untargeted moments				
Share monitoring ($F(\kappa^*)$)		0.04	0.44	0.98
Wage (w)		0.94	0.92	0.89
Job creation (v^n/v)		0.96	0.99	1
Job-finding prob. (p^m)		1	1	0.93
Job-finding prob. (p^w)		0.34	0.25	0

Notes: The low-κ^* steady state is the steady state targeted in calibration. The intermediate-κ^* and high-κ^* steady states are the co-existing but counterfactual steady states associated with the parameters identified through this calibration procedure. See Section 4.1 for details on calibration.

The steady states with higher levels of monitoring feature (i) higher job-finding rates, (ii) lower unemployment, (iii) lower wages and (iv) more job creation. These observations—in particular, the positive correlation between wages and unemployment across steady states—reflect endogenously different levels "labor supply" across steady states, sustained by workers' self-confirming beliefs about the actions of other workers in the economy.

4.3 Welfare

The differences across equilibria reflected in Table 3 translate into differences in social welfare. The social welfare function in the model is given by

$$\Omega = \underbrace{(1-u)y}_{\text{Production}} + \underbrace{u\int_{\kappa^*}^{\infty} \kappa f(\kappa) d\kappa}_{\text{Leisure}} - \underbrace{\left[cv + \tilde{\eta}\frac{\nu}{1+\nu}\left(V^n\right)^{\nu+1}\right]}_{\text{Vacancy costs + Entry costs}}. \tag{46}$$

[19] I measure job creation as the share of all vacancies that are new entrants: v^n/v.

The first term ("Production") corresponds to total output from employed workers matched with entrepreneurs. The second term ("Leisure") corresponds to the value of leisure among workers who choose not to monitor. The third term ("Vacancy costs + Entry Costs") corresponds to the sum of the costs of maintaining a posting for unmatched vacancies and the entry costs for entrepreneurs.

The first term is increasing in κ^* because, as we see in Table 3, unemployment is decreasing in κ^*. The second term, on the other hand, is decreasing in κ^*: More workers choosing to monitor implies both (i) fewer workers unemployed who can enjoy leisure and (ii) a smaller *share* of unemployed workers choosing to enjoy leisure. The final term is less clear: On the one hand, more workers monitoring means that there are fewer vacancies that aren't matched immediately, and thus fewer firms paying costs of maintaining a vacancy (c). On the other hand, more monitoring depresses wages by reducing the (ex ante) value of unemployment, which in turn increases the value to an entrepreneur of posting a new vacancy. This results in more entry from entrepreneurs with higher costs, which detracts from social welfare.

5 Dynamics and Quantitative Implications

I next study the model's local and global dynamics, using the model's global properties illustrate two important implications for aggregate fluctuations in the labor market: First, I show numerically that there are initial unemployment rates for which the economy has equilibria converging to different steady states. This implies that the economy is susceptible to belief-driven fluctuations in labor supply. Second, I show that when a shock—say, to labor demand—causes a sufficiently large increase in unemployment, the economy can be forced onto a unique path towards the high-monitoring steady-state. I argue that this observation can help to explain a number of features of the recovery from the Great Recession.

5.1 Frictionless system ($\psi \to \infty$)

To facilitate exposition of the model's dynamics, I focus on the limiting case in which $\psi \to \infty$. This corresponds to a frictionless aftermarket, implying that $\hat{q}_t = q_t^n = q_t^o = q_t = 1$ and thus that $v_t = v_t^n$. It follows that, in this case, the model has a single endogenous state variable (u_t) and two forward-looking variables (v_t and κ_t^*), which significantly facilitates analyzing the model's global dynamics below. Specifically, it can be shown that the model's equilibria are characterized by three equations:

$$u_{t+1} = u_t + \delta(1 - u_t) - v_t \tag{47}$$

$$v_t = \eta G\left(-c + \beta(1-\delta)\left[y_{t+1} - \omega(\kappa_{t+1}^*, u_{t+1}, v_{t+1})\left(\tfrac{1-(1-b)u_{t+1}}{1-u_{t+1}}\right) + c + G^{-1}\left(\frac{v_{t+1}}{\eta}\right)\right]\right) \tag{48}$$

$$\kappa_t^* = \begin{cases} \beta(1-\delta)\left(\frac{1-v_t/u_t}{1-F(\kappa_t^*)}\right)\frac{\chi}{1-\chi}\left(y_{t+1} - \omega(\kappa_{t+1}^*, u_{t+1}, v_{t+1})\left(\frac{1-(1-b)u_{t+1}}{1-u_{t+1}}\right) + c\right) & \text{if} \quad F(\kappa^*) \le v_t/u_t \\ \beta(1-\delta)\left(\frac{v_t/u_t}{F(\kappa_t^*)}\right)\frac{\chi}{1-\chi}\left(y_{t+1} - \omega(\kappa_{t+1}^*, u_{t+1}, v_{t+1})\left(\frac{1-(1-b)u_{t+1}}{1-u_{t+1}}\right) + c\right) & \text{if} \quad F(\kappa^*) > v_t/u_t \end{cases} \tag{49}$$

where

$$\omega(\kappa_t^*, u_t, v_t) = \begin{cases} \frac{\frac{\chi}{1-\chi}(y_t+c)+\int_{\kappa_t^*}^{\infty}\kappa dF(\kappa)-\kappa_t^*(1-F(\kappa_t^*))}{1-b+\frac{\chi}{1-\chi}\left(\frac{1-(1-b)u_t}{1-u_t}\right)} & \text{if} \quad F(\kappa^*) \leq v_t/u_t \\ \frac{\frac{\chi}{1-\chi}(y_t+c)+\int_{\kappa_t^*}^{\infty}\kappa dF(\kappa)-\left(\frac{u_t-v_t}{v_t}\right)\kappa_t^* F(\kappa_t^*)}{1-b+\frac{\chi}{1-\chi}\left(\frac{1-(1-b)u_t}{1-u_t}\right)} & \text{if} \quad F(\kappa^*) > v_t/u_t. \end{cases} \tag{50}$$

The frictionless system characterized by equations (47)-(50) continues to have three steady-state equilibria, as in the full quantitative model described in Section 3. Furthermore, these three steady states retain the same qualitative features of those in Section 3. I thus proceed with an analysis of the frictionless model.

5.2 Dynamics

With equations (47)-(49) in hand, we are prepared to study the model's dynamic properties. I first analyze the model's *local* dynamics in a neighborhood around each of the three steady-state equilibria. I then proceed to analyze the model's *global* dynamics away from these steady states.

5.3 Local dynamics

To analyze the model's local dynamics, I linearize the system in (47)-(49) around each of the three steady states. The linearized system of equations is given by:

$$\begin{bmatrix} u_{t+1} \\ v_{t+1} \\ \kappa_{t+1}^* \end{bmatrix} = \begin{bmatrix} u \\ v \\ \kappa^* \end{bmatrix} + \begin{bmatrix} \frac{\partial u_{t+1}}{\partial u_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial u_{t+1}}{\partial v_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial u_{t+1}}{\partial \kappa_t^*}\big|_{\{u,v,\kappa^*\}} \\ \frac{\partial v_{t+1}}{\partial u_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial v_{t+1}}{\partial v_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial v_{t+1}}{\partial \kappa_t^*}\big|_{\{u,v,\kappa^*\}} \\ \frac{\partial \kappa_{t+1}^*}{\partial u_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial \kappa_{t+1}^*}{\partial v_t}\big|_{\{u,v,\kappa^*\}} & \frac{\partial \kappa_{t+1}^*}{\partial \kappa_t^*}\big|_{\{u,v,\kappa^*\}} \end{bmatrix} \begin{bmatrix} u_t - u \\ v_t - v \\ \kappa_t^* - \kappa^* \end{bmatrix} \tag{51}$$

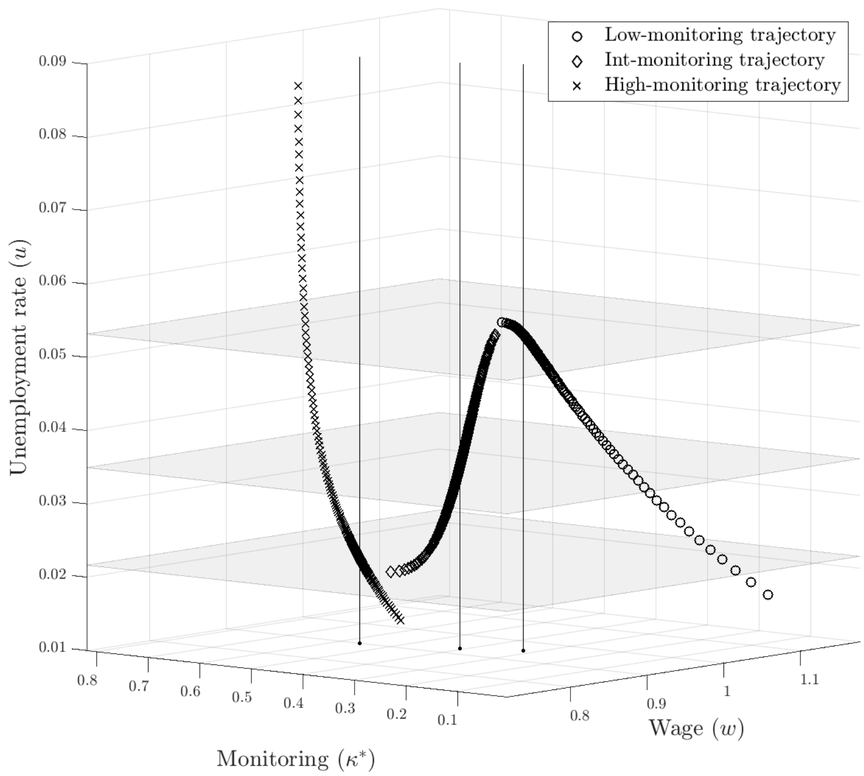
The Jacobian in (51) determines the system's local dynamics around each of the three steady states. Because it is not possible to solve for the elements of the Jacobian analytically, I do so numerically. Based on the parameterization in Section 4.1 (with $\psi \to \infty$), the Jacobian in (51) has two eigenvalues outside of the unit circle and one eigenvalue inside of the unit circle *for all three steady states*. Because the model has one predetermined variable (u_t) and two forward-looking variables (v_t and κ_t^*), this implies that a unique non-explosive solution exists for all three steady states. This implies that, if we momentarily neglect the multiplicity of steady states, given some initial condition u_0 in a neighborhood of any of the steady states, κ_t^* and v_t will immediately jump to be on the saddle path that leads the economy back to the steady state.

5.4 Global dynamics

Knowledge of the model's local dynamics as discussed above can be used to construct global stable manifolds and thus to study the model's global dynamics. Specifically, following Judd (1999) and Brunner and Strulik (2002), I numerically construct the global stable manifolds converging to the three steady states using backward integration. In effect, this method takes advantage of the fact that, when we solve the system in reverse time, the stable manifolds in forward time become the unstable manifolds. Paths attracted to the unstable manifold from which we necessarily begin thus feature "exponential tracking"—i.e., exponentially decreasing deviations from the stable manifold as we run the system in reverse time.[20]

Figure 2 plots the three global stable manifolds computed in this way. The grey planes depict the three steady-state levels of unemployment. The vertical lines depict the three steady-state pairs of values $\{\kappa^*, w\}$. Figure 2 highlights two central features of the model. First, there can be

Figure 2: Global dynamics



Notes: Equilibrium trajectories converging towards the three steady states.

multiple equilibria leading to different steady states. Second, for sufficiently high initial values of unemployment, there is a unique equilibrium that converges to the high-monitoring steady-state.

5.5 Belief-driven labor supply fluctuations

In Section 4.1, we observed that the economy has multiple steady-state equilibria (which continues to be the case as $\psi \to \infty$). Figure 2 illustrates that the economy also has multiple *dynamic* equilibria. More precisely, there is a range of initial conditions for the economy, i.e., $u_0 \in [\underline{u}, \bar{u}]$, for which the economy can converge to any of the three steady states. Indeed, inspection of (2) reveals that this is possible from any of the steady states and from any u_0 between any of these

[20]See Atolia and Buffie (Atolia and Buffie) for a good discussion of this and solution methods for models with two or more state variables.

steady states. Within this range, then, which equilibrium is not pinned down by fundamentals but rather determined by workers' beliefs about the intensity with which other workers in the economy are monitoring the arrival of vacancies. Put differently, individual

Functionally, the multiplicity of dynamic equilibria observed in Figure 2 implies that the economy is susceptible to extrinsic shocks to workers' beliefs that can permanently alter the trajectory of the economy by shifting the economy from a path leading to one steady state to a path leading to another. Because such changes in beliefs manifest through unemployed workers' decisions to actively monitor the arrival of vacancies, the model can be understood as providing a theory of endogenous—belief-induced—labor supply fluctuations. In Section 6, I provide some suggestive evidence that, in the years since online job search has become widespread, labor supply shocks have grown in importance as a source of fluctuations for the U.S. economy.

5.6 The Great Recession

Figure 2 reveals a second property of the model: When the unemployment rate is sufficiently high, there is only one dynamic equilibrium—the one leading to the high-monitoring steady state. This observation echoes Proposition 1 from Section 2, which established that, when the economy is sufficiently slack, there is a unique equilibrium corresponding to a high level of monitoring. Whereas the discussion above is important because it implies that the economy can be permanently affected by non-fundamental changes in beliefs, the at high values of unemployment observed in Figure 2 is important because it implies that *fundamental* shocks can force the economy out of a particular equilibrium and into another.

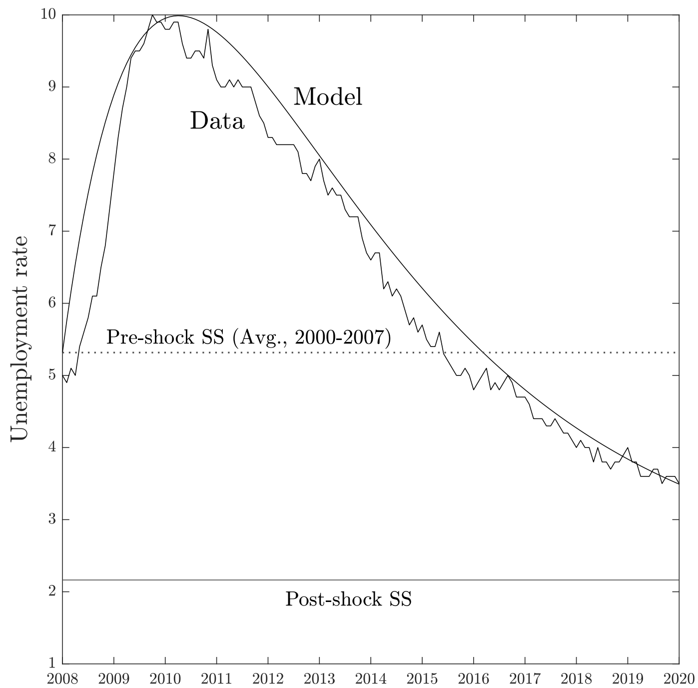
While this observation is interesting in its own right, it turns out that it also offers a novel explanation for several difficult-to-explain features of the prolonged recovery from the Great Recession. To illustrate this point parsimoniously and in a way that is consistent with the financial turmoil that preceded the Great Recession, I consider the implications of a large and persistent shock to the cost of financing investments in job creation among entrepreneurs, $\tilde{\eta}$.[21] While this is not intended to be a quantitative exercise, it is a simple way to qualitatively explain several facts about the recovery from the Great Recession.

5.6.1 Unemployment

Figure 3 depicts the effect of the entrepreneurial shock on the unemployment rate in the model, and compares the model's response with the data. The dashed horizontal line depicts the average unemployment rate prior to the Great Recession (2000-2007) to which the model is calibrated. The figure is intended to make a simple point: A shock that causes unemployment to rise to 10% *necessarily* causes the economy to jump to an equilibrium trajectory that will ultimately lead to a *lower unemployment rate than that from which the economy began.* This implication, of course, is consonant with the experience of the ten years following the financial crisis and Great Recession, during which unemployment fell to below 4% prior to the Covid-19 crisis. Standard macroeconomic models with unique steady states cannot easily account for this fact.

[21] Recall from Section 3 that, in order to undertake an investment project and thus create a job, an entrepreneur must pay a sunk investment cost drawn from distribution G with support $[0, \bar{\xi}]$. Alternatively, this can simply be interepreted as a shock to the measure of entrepreneurs in the economy emanating from some other source.

Figure 3: Unemployment during the Great Recession



Notes: Pre-shock SS corresponds to the low-monitoring steady-state calibrated to match the unemployment rate between 2000 and 2007. Post-shock SS corresponds to the (untargeted) high-monitoring steady-state.

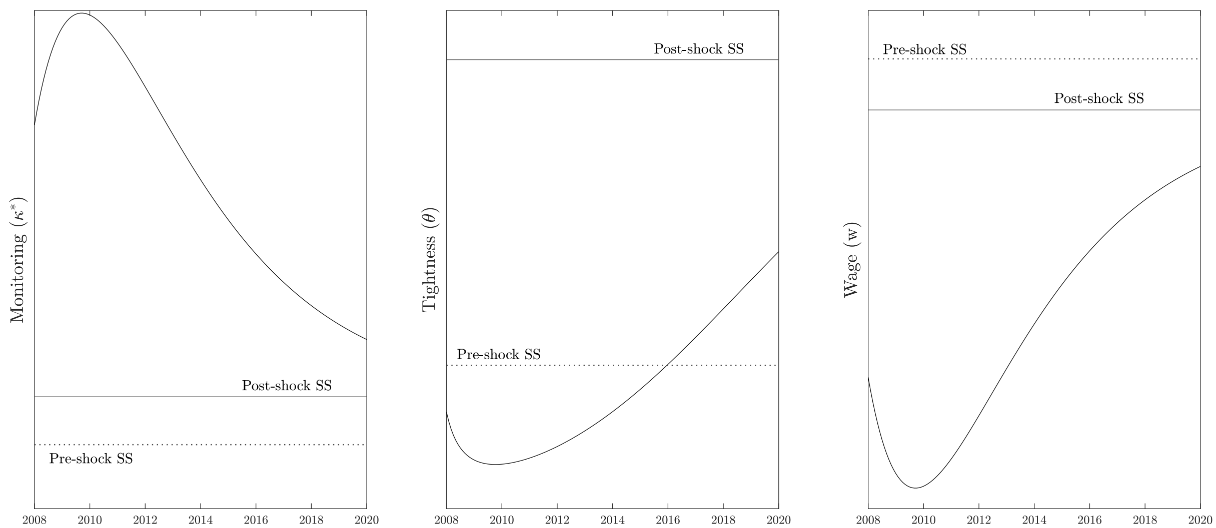
5.6.2 Wages and tightness

On November 5, 2019—almost exactly ten years after unemployment reached 10% in October 2009—John Robertson posted on the Atlanta Fed's MacroBlog about the state of the labor market ten years into the recovery. Robertson writes:

> *Here's a puzzle. Unemployment is at a historically low level, yet nominal wage growth is not even back to prerecession levels (see, for example, the Atlanta Fed's own Wage Growth Tracker). Why is wage growth not higher if the labor market is so tight?*

This quote captures two additional features of the recovery from the Great Recession that are perplexing from the standpoint of traditional models: The labor market (ultimately) was quite tight, but wage growth was persistently tepid. In the presence of a large initial shock such as the one described above, the model considered in this paper can qualitatively account for both of these observations. Figure 4 illustrates this point, plotting the (unique) model-implied trajectories of monitoring intensity, tightness and wages.

Following the shock, the economy is forced into the high-monitoring equilibrium, which *permanently* depresses wages by reducing the expected value of unemployment. Wages eventually begin to recovery, but only to the high-monitoring steady state at which they are lower than they were prior to the shock. This weak recovery in wages, in turn, gives rise to a strong recovery vacancies, which recover to a level that exceeds their level prior to the shock. Together with low (and falling) unemployment, this implies that the economy was on a trajectory converging to an historically



(a) Monitoring (b) Tightness (c) Wages

Figure 4: The recovery from the Great Recession

tight labor market. Thus, the model accounts for the co-existence of tepid wage growth in a tight labor market. Of course, these observations would be difficult to understand in the context of a demand-driven explanation, even one featuring the possibility of multiple equilibria, since the strong demand required to generate a tight labor market would necessarily lead also, and counterfactually, to permanently elevated wages.

6 Some Evidence on Monitoring and its Implications

Finally, I provide two suggestive pieces of evidence in support of, in turn, the mechanism and its implications. First, using high-frequency panel data on unemployed workers' search behavior, I show that online job search is significantly less lumpy than traditional job search. This is consistent with online job search facilitating monitoring the arrival of new job openings. Second, I show that the contribution of labor supply shocks to aggregate fluctuations increased markedly around the turn of the century. This is consistent with the emergence of endogenous belief-driven fluctuations in job search driven by the rapid growth in online search as discussed above.

6.1 Micro evidence: The internet makes search less lumpy

The mechanism considered in this paper is predicated on the idea that online job search reduces the fixed costs of checking for new job listings, thereby allowing workers to frequently do so and, in effect, to continuously monitor new listings.[22] High fixed costs of frequently checking for new listings, such as we should expect to be associated with the pre-online job search era, make monitoring infeasible, thus rendering individuals' job search decisions relatively "lumpy" in the sense of occurring infrequently but for long periods to economize on fixed costs. Evidence that online

[22] See the introduction for examples of technologies that reduce fixed costs an enable monitoring.

search is less lumpy that offline job search would thus constitute evidence in support of the central hypothesis of this paper regarding the mode by which technology affects search behavior.

Unfortunately, the vast majority of data on job search is simply too low frequency to make any meaningful determinations about whether online job search is less lumpy than regular job search at a relevant frequency. Moreover, data on whether individuals are using the internet for job search is both relatively scarce and likely to be contaminated by non-trivial selection effects. To overcome these empirical challenges, I leverage data from the Survey of Unemployed Workers in New Jersey (SUWNJ) to attempt to bring some new evidence to bear on the question of whether the internet makes search less lumpy. The SUWNJ is a weekly longitudinal survey, lasting up to 24 weeks, of 6,025 unemployment insurance benefit recipients in New Jersey, that was conducted between the fall of 2009 and early 2010.[23] Importantly, the survey contains data on job search behavior, both in the form of weekly recall questions about activity over the entire previous week, as well as a once-weekly time diary. Most relevantly for studying whether the internet makes search less lumpy, the SUWNJ contains: (i) a question asking respondents whether they used the internet to search for work in the past week, (ii) a question asking respondents how many hours they spent searching in the past week, and (iii) a time diary that can be used to determine whether or not a respondent searched on the day on which they filled out the diary.

The motivation for the empirical strategy and specification below is straightforward: If online job search is smoother than offline job search for reasons described above, then the probability that a respondent searches on any given day should be higher for a person who uses the internet for job search than for a person who does not, all else equal. Thus, we can regress an indicator for whether a respondent reported job search on a random day on (i) whether they reported using the internet for search over the past 7 days and (ii) how much they searched over the past 7 days (to control for how much the person is searching in general).

Following this logic, let s_{it}^{TD} denote total time spent on search by respondent i in week t based on the time diary data (i.e., on a single day of the week), s_{it}^{WR} the weekly recall measure of total time spent on search in the past 7 days, $\mathbb{1}(\text{OJS}_{it})$ a dummy for whether a respondent reports having used the internet for job search in the past 7 days, τ_t a calendar week dummy, and η_i a respondent fixed effect. Then, I estimate a simple linear probability model of the form:

$$Pr(s_{it}^{\text{TD}} > 0) = \alpha + \beta s_{it}^{\text{WR}} + \gamma \mathbb{1}(\text{OJS}_{it}) + \tau_t + \eta_i + \epsilon_{it}. \tag{52}$$

Table 4 reports the main results. The results indicate that reporting engaging in online search increases the probability of searching on the previous day by at least 13 percentage points. This is consistent with the notion that online searchers smooth out their search across days, one form of which is monitoring the arrival of vacancies. While I do not present other robustness checks here, the results are robust to a variety of different specifications and inclusion of additional controls.

6.2 Macro evidence: Labor supply shocks are increasingly important

Macroeconomists have long been interested in identifying the contribution of labor supply shocks to aggregate fluctuations. This task is approached in the SVAR literature by proposing theoretically motivated restrictions that enable labor supply shocks to be plausibly disentangled from other

[23] Complete survey data and documentation can be obtained from: https://dss.princeton.edu/catalog/resource1350.

Table 4: Smooth search

	Spec. 1	Spec. 2	Spec. 3
Online Search	0.16***	0.16***	0.13***
	(0.04)	(0.05)	(0.04)
Total Search	0.03***	0.03***	0.03***
	(0.01)	(0.01)	(0.01)
Total App'ns		0.07***	0.06***
		(0.01)	(0.01)
Calendar time:	×	×	×
Ind. FE:	×	×	×
Day of week:			×
Observations	7,705	7,328	7,328

Robust standard errors in parentheses.

Notes: Sample restricted to respondents who have never accepted a job offer and who are between the ages of 20 and 60. All regressions use survey weights.

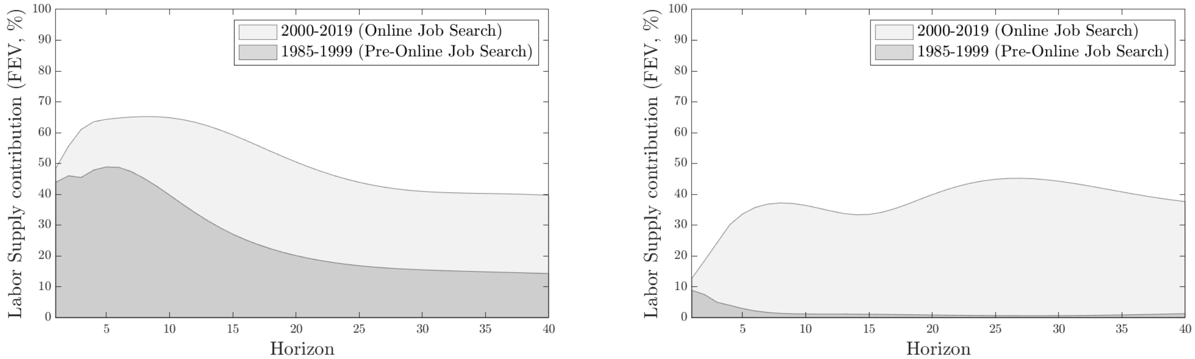
sources of aggregate fluctuations.[24] The model described in this paper can be understood as a theory of (endogenous) labor supply shocks. Given the close link between the underlying mechanism—monitoring—and the advent of online job search, it is natural to ask whether labor supply shocks have contributed more to aggregate fluctuations in the past 20 years than they did previously.

Accordingly, I estimate a standard three-variable VAR using a sign restriction approach similar to that in Peersman and Straub (2009) and further refined by Foroni et al. (2018). Specifically, I estimate a VAR with output (quarterly real output in the non-farm business sector from the BLS), the price level (GDP deflator), and a measure of real wages (total private average hourly earnings of production and non-supervisory employees, deflated by the GDP deflator), each measured at a quarterly frequency. The VAR includes four lags and is estimated in (log) levels. Following Foroni et al. (2018), I assume that (i) aggregate demand shocks move output and the price level in the same direction, and (ii) technology shocks move output and the price level in opposite directions (thus identifying them from aggregate demand shocks) and move the real wage in the opposite direction as labor supply shocks. Belief-generated shifts between the high- and low-monitoring steady states will satisfy these restrictions.

Reflecting the role of the internet in enabling job seekers to monitor vacancies, I split the post-1985 sample into two non-overlapping periods: 1985Q1-1999Q4 and 2001Q1-2019Q4. I then estimate the VAR separately on each sample and report the implied forecast error variance decomposition. Figure 5 reports the results for output and wages at horizons of up to 10 years (40 quarters).

Panel 5a reports the contribution of labor supply shocks to variation in output and Panel 5b reports the contribution of labor supply shocks to variation in the real wage. In both cases, we see that the contribution of labor supply shocks is markedly higher in the past 20 years than it was in the

[24]See, for example, Blanchard and Diamond (1989), Chang and Schorfheide (2003), Peersman and Straub (2009), and Foroni et al. (2018).



(a) Output

(b) Wage

Figure 5: Variance decomposition: Pre- and post-online job search

preceding 15, prior to the advent of online job search. Furthermore, this observation holds at all horizons reported. While not conclusive, this evidence is suggestive of a role for the theory of labor supply shocks proposed in this paper.

7 Conclusion

This paper develops a parsimonious model of search as monitoring and studies its implications for equilibrium labor market dynamics.

References

Anderson, P. and B. Meyer (1997). Unemployment insurance takeup rates and the after-tax value of benefits. *Quarterly Journal of Economics*.

Atolia, M. and E. Buffie. Reverse shooting made easy: Automating the search for the global nonlinear saddle path. Manuscript.

Barron, J., M. Berger, and D. Black (1997). Employer search, training, and vacancy duration. *Economic Inquiry*.

Beaudry, P., D. Green, and B. a. Sand (2018). In search of labor demand. *American Economic Review*.

Belot, M., P. Kircher, and P. Muller (2019). Providing advice to job seekers at low cost: An experimental study on online advice. *Review of Economic Studies*.

Blanchard, O. and P. Diamond (1989). The beveridge curve. *Brookings Papers on Economic Activity*.

Brunner, M. and H. Strulik (2002). Solution of perfect foresight saddle point problems: a simple method and applications. *Journal of Economic Dynamics and Control*.

Cairo, I., S. Fujita, and C. Morales-Jimenez (2021). The cyclicality of labor force participation flows: The role of labor supply elasticities and wage rigidity. *Review of Economic Dynamics*.

Carnevale, A., T. Jayasundera, and D. Repnikov (2014). Understanding online job ads data. Technical report, Georgetown University.

Chang, Y. and F. Schorfheide (2003). Labor-supply shifts and economic fluctuations. *Journal of Monetary Economics*.

Chodorow-Reich, G. and L. Karabarbounis (2016). The cyclicality of the opportunity cost of employment. *Journal of Political Economy*.

Coles, M. and A. Moghaddasi Kelishomi (2018). Do job destruction shocks matter in the theory of unemployment? *American Economic Journal: Macroeconomics*.

Davis, S. and B. Samaniego de la Parra (2020). Application flows. *Working paper*.

Diamond, P. (1982a). Aggregate demand management in search equilibrium. *Journal of Political Economy 90*(5).

Diamond, P. (1982b). Wage determination and efficiency in search equilibrium. *Review of Economic Studies*.

Faberman, J. and M. Kudlyak (2016). What does online job search tell us about the labor market? *Federal Reserve Bank of Chicago Economic Perspectives*.

Faberman, J. and G. Menzio (2018). Evidence on the relationship between recruiting intensity and starting wage. *Labour Economics*.

Foroni, C., F. Furlanetto, and A. Lepetit (2018). Labor supply factors and economic fluctuations. *International Economic Review*.

Fujita, S. and G. Ramey (2005). The dynamic beveridge curve. *Federal Reserve Bank of Philadelphia Working Papers*.

Fujita, S. and G. Ramey (2012). Exogenous versus endogenous separation. *American Economic Journal: Macroeconomics*.

Hall, R. and M. Kudlyak (2022). The inexorable recoveries of unemployment. *Working paper*.

Hall, R. and P. Milgrom (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*.

Judd, K. (1999). *Numerical Methods in Economics*. MIT Press.

Krusell, P., T. Mukoyama, R. Rogerson, and A. Sahin (2017). Gross worker flows over the business cycle. *American Economic Review*.

Kuhn, P. and H. Mansour (2013). Is internet job search still ineffective? *The Economic Journal*.

Menzio, G. and P. Martellini (2020). Declining search frictions, unemployment and growth. *Journal of Political Economy 128*.

Mortensen, D. (1982). The matching process as a noncooperative bargaining game. In U. of Chicago Press (Ed.), *The Economics of Information and Uncertainty*.

Mueller, A., D. Osterwalder, J. Zweimuller, and A. Kettemann (2020). Vacancy durations and entry wages: Evidence from linked vacancy-employer-employee data. *Unpublished manuscript*.

Peersman, G. and R. Straub (2009). Technology shocks and robust sign restrictions in a euro area svar. *International Economic Review*.

Pissarides, C. (1985). Short-run dynamics of unemployment, vacancies, and real wages. *American Economic Review*.

Pries, M. and R. Rogerson (2022). Declining worker turnover: The role of short-duration employment spells. *American Economic Journal: Macroeconomics*.

Shimer, R. (2005). The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*.

Shimer, R. (2012). Reassessing the ins and outs of unemployment. *Review of Economic Dynamics*.

van Ours, J. and G. Ridder (1992). Vacancies and the recruitment of new employees. *Journal of Labor Economics*.

Appendices

A Monitoring game

A.1 Proposition 1: Necessary and sufficient conditions for multiplicity.

Proof.

Necessary condition: The proof proceeds by showing that there exists a value $\underline{\theta}$ such that for any $\theta < \underline{\theta}$ there must exist a unique equilibrium. Define $\underline{\theta}_0 \equiv \min\{\frac{\bar{\kappa}}{w-b}, F(w-b)\}$. Because $\bar{\kappa} < w - b$ by assumption, $\frac{\bar{\kappa}}{w-b} < 1$. Furthermore, because F is a distribution function and $\underline{\kappa} < \bar{\kappa} < w - b$, it must be that $F(w-b) \in (0, 1]$. Thus, $\underline{\theta}_0 \in (0, 1]$. To establish existence of an equilibrium, because κ^U is decreasing on $[\kappa^c, \bar{\kappa}]$, it is sufficient to establish that (i) $\frac{\theta(w-b)}{F(\bar{\kappa})} \leq \bar{\kappa}$ and (ii) $\frac{\theta(w-b)}{F(\kappa^c)} \geq \kappa^c$. It is easy to verify that these two conditions hold for any $\theta < \bar{\theta}_0$. To establish uniqueness, note in the limit as $\theta \to 0$, the only possible equilibrium such that $\theta \geq F(\kappa^*)$ corresponds to $\kappa^* = \underline{\kappa}$, which, by (10), requires $w - b < \underline{\kappa} < \bar{\kappa}$, a contradiction. Thus, there exists some $\underline{\theta} < \underline{\theta}_0$ such that there is a unique equilibrium for any $\theta < \underline{\theta}$. Furthermore, that equilibrium must be such that $\theta < F(\kappa^*)$.

Sufficient condition: I first prove that for any $\theta > 1/2$, there are multiple equilibria provided search cost are bounded between $[(1-\theta)(w-b), \theta(w-b)]$. From there, it follows immediately from the assumption that $\bar{\kappa} < w - b$ that there must exist some $\bar{\theta}$ such that for any $\theta > \bar{\theta}$, there is multiplicity. The interval $[(1-\theta)(w-b), \theta(w-b)]$ is non-empty only if $\theta > \frac{1}{2}$. Suppose $\kappa^* = \underline{\kappa}$ so that $F(\kappa^*) = 0$ and no workers search. Then, using (9) and the fact that $F(\kappa^*) = 0 < \theta$, $\underline{\kappa} > (1-\theta)(w-b)$ implies that $\kappa^*(\kappa^*) = \kappa^L(\kappa^*) = \underline{\kappa}$, so $\kappa^* = \underline{\kappa}$ is an equilibrium. Suppose instead that $\kappa^* = \bar{\kappa}$ so that $F(\kappa^*) = 1$ and all workers search. Then, again using (9) and the fact that $F(\kappa^*) = 1 > \theta$, $\bar{\kappa} < \theta(w-b)$ implies that $\kappa^*(\kappa^*) = \kappa^H(\kappa^*) = \bar{\kappa}$, so $\kappa^* = \bar{\kappa}$ is an equilibrium. Thus, there are multiple equilibria. □

A.2 Proposition 2: Impossibility of multiplicity in traditional model

Proof. From the best-response function in (3), a necessary condition for multiplicity is that $p^h - p^l$ is increasing in κ^* (that is, increasing returns are necessary for multiplicity). Because $z < 1$, (12) implies that $\tilde{s}(\kappa^*)$ is increasing in κ^*, while (14) and (15) imply that $p^h - p^l$ is decreasing in $\tilde{s}(\kappa^*)$. Thus, $p^h - p^l$ is decreasing in κ^*, so there cannot be multiple equilibria. □

B Traditional model

This appendix describes a traditional model of matching reflecting how search is typically modeled in the literature—as a decision concerning the intensity of effort rather than as a decision concerning whether to monitor new jobs.

B.1 Search as variable effort

The traditional approach to modeling job search decisions in quantitative macroeconomic models is to assume that searchers and non-searchers are perfect substitutes in the match function. This implies that non-searchers are simply ineffective searchers, matching at a rate proportional to that of searchers. Recent examples of such models include Krusell et al. (2017) and Cairo et al. (2021). The description of search and matching below reflects this interpretation.

Consider a model in which workers choosing to search exert one unit of effort, whereas workers choosing not to search exert $z < 1$ units of effort. "Effort" here can be interpreted as the probability of participating in the market. Taking this interpretation and dropping time subscripts for ease of notation, we can define the effective measure of searchers in the economy, s, as

$$
\begin{aligned}
s &= F(\kappa^*)u + (1 - F(\kappa^*))zu && \text{(B.1)}\\
&= \Big[F(\kappa^*) + (1 - F(\kappa^*))z\Big]u && \text{(B.2)}\\
&= \tilde{s}u. && \text{(B.3)}
\end{aligned}
$$

where $\tilde{s}$ can be interpreted as average search effort in the economy.

B.2 Monitoring Game

The traditional characterization of search described above can be immediately embedded in the basic environment described in Section 2. Specifically, the search decision described in Section 2.1.1 does not depend on interpreting search as monitoring, and thus continues to hold when search is interpreted instead as a choice of intensity. The only change, then, are the match rates in equations (6) and (7). In Section 2, I discuss how the changes implied by this more traditional model of search affect the existence of increasing returns and thus multiple equilibria.

B.3 Dynamic model

Adopting the functional form of the match function from Section 3, the total number of matches in this economy is given by

$$m_t = \mu(v_t, s_t) = v_t \left[1 - e^{-\psi \frac{s_t}{v_t}}\right]. \tag{B.4}$$

The implied match rates for searchers (high-effort) and non-searchers (low-effort), respectively, are thus

$$
\begin{aligned}
p_t^h &= \frac{v_t}{s_t}\left[1 - e^{-\psi \frac{s_t}{v_t}}\right] && \text{(B.5)}\\
p_t^l &= z\frac{v_t}{s_t}\left[1 - e^{-\psi \frac{s_t}{v_t}}\right]. && \text{(B.6)}
\end{aligned}
$$

The average match rate for workers, in turn, is

$$p_t = F(\kappa_t^*)p_t^h + (1 - F(\kappa_t^*))p_t^l \tag{B.7}$$

$$= \theta_t \left[1 - e^{-\psi \frac{\tilde{s}_t}{\theta_t}}\right] \tag{B.8}$$

where the second line uses the expression for s_t in (B.3), (B.5), (B.6), and $\theta_t \equiv v_t/u_t$. Thus, changes in average search effort are isomorphic to changes in ψ, the parameter governing frictions in the matching process. Note that this implies that we can write the difference between the match rate associated with searching and the match rate associated with not searching (which governs the search decision) as

$$p_t^h - p_t^l = \frac{1-z}{\tilde{s}_t} p_t. \tag{B.9}$$

Finally, the average match rate for firms in the traditional model is just

$$q_t = p_t/\theta_t = 1 - e^{-\psi \frac{\tilde{s}_t}{\theta_t}}. \tag{B.10}$$

The foregoing can be used to calibrate the model using the exact same moments that are used to calibrate the model in Section 4.1 as described in Appendix C.

C Calibration

This appendix provides further details on the calibration of ψ, ν, $\tilde{\eta}$, μ_κ, and σ_κ. All other parameters are calibrated directly to match values in the data and existing literature as described in Section 4.1. The model is calibrated under an assumption that the data used for calibration was generated by an economy in the vicinity of a steady state in which not all vacancies match during the monitoring phase.

Match efficiency (ψ). I first calibrate the aftermarket match efficiency parameter (ψ). Because not all vacancies match during the monitoring phase, i.e., $v^n > u^m$, (20) implies that the number of matches in the monitoring phase is

$$\mathring{m} = u^m = F(\kappa^*)u. \tag{C.1}$$

This implies that the fraction of all vacancies that match during the monitoring phase, $\mathring{m}/v$, can be written as $F(\kappa^*)u/v = F(\kappa^*)/\theta$. I assume that postings matched during the monitoring phase are active for less than 48 hours while all other postings are active for over 48 hours, which implies that $F(\kappa^*)/\theta$ corresponds to the proportion of vacancy postings lasting for under 48 hours, Γ^m:

$$\Gamma^m \equiv F(\kappa^*)/\theta. \tag{C.2}$$

As discussed in the main text, Davis and Samaniego de la Parra (2020) identify a value of $\Gamma^m = 0.2$. Using this together with the long-run average value of market tightness, $\theta = 0.52$, we can solve for the fraction of unemployed workers who monitor vacancies,

$$F \equiv F(\kappa^*) = \theta \cdot \Gamma. \tag{C.3}$$

The preceding implies that the number of matches in the aftermarket, determined by (45), is

$$\hat{m} = (v - Fu)\left[1 - e^{-\psi\frac{u-Fu}{v-Fu}}\right] \tag{C.4}$$

$$= u(\theta - F)\left[1 - e^{-\psi\frac{1-F(\kappa^*)}{\theta - F(\kappa^*)}}\right]. \tag{C.5}$$

Thus, the total number of matches is

$$m = \mathring{m} + \hat{m} \tag{C.6}$$

$$= Fu + u(\theta - F)\left[1 - e^{-\psi\frac{1-F}{\theta-F}}\right] \tag{C.7}$$

$$= u\left(F + (\theta - F)\left[1 - e^{-\psi\frac{1-F}{\theta-F}}\right]\right) \tag{C.8}$$

and the average match rate for vacancies, q, is

$$q = m/v \tag{C.9}$$

$$= F/\theta + (1 - F/\theta)\left[1 - e^{-\psi\frac{1-F}{\theta-F}}\right]. \tag{C.10}$$

Note that we can use the average monthly hazard rate from Shimer (2005), $p = 0.45$, together with the long-run value for market tightness above, to solve for q:

$$q = p/\theta. \tag{C.11}$$

This allows us to solve (C.10) for the match efficiency parameter, ψ:

$$\psi = -\frac{\theta - F}{1 - F} \ln \left(1 - \frac{q - F/\theta}{1 - F/\theta}\right). \tag{C.12}$$

Job creation ($\tilde{\eta}$, ν). I next calibrate the parameter scaling the vacancy-creation process ($\tilde{\eta}$) and the parameter governing the elasticity of the vacancy-creation process (ν). To do so, it is first necessary to solve for the steady-state value of a new vacancy.

The law of motion for unemployment in (18) implies that the steady-state level of unemployment (equivalently, the steady-state unemployment rate) is

$$u = \frac{\delta}{\delta + (1 - \delta)p}. \tag{C.13}$$

Using the definition of market tightness, $\theta \equiv v/u$, and its long-run value given above, the steady-state level of unemployment pins down the steady-state level of vacancies,

$$v = \theta u. \tag{C.14}$$

Since we have already solved for the steady-state average match rate for vacancies, the law of motion for vacancies in (19) then pins down the steady-state level of new vacancies via

$$v^n = v(1 - (1 - \delta)(1 - q)). \tag{C.15}$$

From here, we can solve for all of the relevant match rates. Specifically, since we now know u, F and ψ (and since θ is chosen to match its empirical value), equations (C.1), (C.5), and (C.8) pin down steady-state values for the number of matches in the monitoring phase, the number of matches in the aftermarket, and the total number of matches. Using these and equations (22) and (25), the steady-state match rates for vacancies in the monitoring phase and aftermarket, respectively, are

$$\mathring{q} = \mathring{m}/v^n \tag{C.16}$$

$$\hat{q} = \hat{m}/(v - \mathring{m}) \tag{C.17}$$

and, from (40) and (41), the steady-state match rates for newly posted vacancies and old vacancies, respectively, are

$$q^n = \mathring{q} + (1 - \mathring{q})\hat{q} \tag{C.18}$$

$$q^o = \hat{q}. \tag{C.19}$$

Likewise, from equations (21) and (24), the steady-state match rates for workers in the monitoring phase and aftermarket, respectively, are

$$\mathring{p} = \mathring{m}/u^m \tag{C.20}$$
$$\hat{p} = \hat{m}/(u - \mathring{m}) \tag{C.21}$$

and, from (27) and (28), the steady-state match rates for workers who choose to monitor and who chose not to monitor, respectively, are

$$p^m = \mathring{p} + (1 - \mathring{p})\hat{p} \tag{C.22}$$
$$p^w = \hat{p}. \tag{C.23}$$

Next observe that in the model, the average value flow value of non-employment is

$$E[z] \equiv bw + \int_{\kappa^*}^{\infty} \kappa dF(\kappa) \tag{C.24}$$

where the first term is the UI payment (where b is the replacement rate) and the second term is the average value of leisure or home production across all non-employed workers. Hall and Milgrom (2008) compute a value corresponding to $E[z] = 0.7$, which is also in the middle of the range of values of the opportunity cost of employment identified by Chodorow-Reich and Karabarbounis (2016). Furthermore, the Nash bargaining solution in (42) implies a wage of

$$w = \frac{\left(\frac{\chi}{1-\beta(1-\delta)(1-q^o)}\right)(y+c) + \left(\frac{1-\chi}{1-\beta(1-\delta)(1-p)}\right)\int_{\kappa^*}^{\infty} \kappa dF(\kappa)}{\left(\frac{1-(1-b)u}{1-u}\right)\left(\frac{\chi}{1-\beta(1-\delta)(1-q^o)}\right) + (1-b)\left(\frac{1-\chi}{1-\beta(1-\delta)(1-p)}\right)}. \tag{C.25}$$

Using both (C.24) and (C.25), we can eliminate the partial expectation $\int_{\kappa^*}^{\infty} \kappa dF(\kappa)$ (which depends on parameters of the distribution of κ which we have not yet calibrated) and solve for the steady-state wage as a function of b and $E[z]$, both of which we know:

$$w = \frac{\left(\frac{\chi}{1-\beta(1-\delta)(1-q^o)}\right)(y+c) + \left(\frac{1-\chi}{1-\beta(1-\delta)(1-p)}\right)E[z]}{\left(1 + b\left(\frac{\frac{1-\chi}{1-\beta(1-\delta)(1-p)}}{\left(\frac{1-(1-b)u}{1-u}\right)\frac{\chi}{1-\beta(1-\delta)(1-q^o)} + (1-b)\frac{1-\chi}{1-\beta(1-\delta)(1-p)}}\right)\right)\left[\left(\frac{1-(1-b)u}{1-u}\right)\frac{\chi}{1-\beta(1-\delta)(1-q^o)} + (1-b)\left(\frac{1-\chi}{1-\beta(1-\delta)(1-p)}\right)\right]}. \tag{C.26}$$

Now that we have a value for w, we can solve for the value of a filled job using (39),

$$J = \frac{y - w - \tau}{1 - \beta(1-\delta)} \tag{C.27}$$

which in turn allows us to solve for the value of an old vacancy and the value of a new vacancy, respectively

$$V^o = \frac{-c + \beta(1-\delta)q^o J}{1 - \beta(1-\delta)(1-q^o)} \tag{C.28}$$
$$V^n = -c + \beta(1-\delta)\left[q^n J + (1-q^n)V^o\right]. \tag{C.29}$$

Finally, to calibrate the two parameters governing the vacancy-creation process, I use the steady-state version of the vacancy-creation condition in (36) and (44):

$$v^n = \tilde{\eta}(V^n)^\nu \tag{C.30}$$

where $\tilde{\eta} \equiv \eta\bar{\xi}^{-\nu}$.[25]

I calibrate ν to match estimates of the wage elasticity of labor demand from the literature. Specifically, I choose ν so that the model yields a steady-state elasticity

$$\epsilon_w^e \equiv \frac{de}{dw} \cdot \frac{w}{e} \tag{C.31}$$

that is consistent with empirical studies.

Leisure/home production distribution (μ_κ, σ_κ). Finally, I calibrate the two parameters of log-Normal distribution from which unemployed workers choosing not to monitor draw flow utility κ_{it} (e.g., leisure or home production). To proceed, first observe that the steady state wage and match rates computed above allow us to solve for workers' value functions in (29), (30), (31), and (32), which in turn allow us to solve for the cutoff value of κ_{it}, κ^*, that determines which workers monitor and which do not. Using (33), we have

$$\kappa^* = \beta(1-\delta)(p^m - p^w)[W - U]. \tag{C.32}$$

From here, I use two restrictions to tie down μ_κ and σ_κ: The first is the restriction that the steady-state share of unemployed workers choosing to monitor—F, the steady-state value of which is tied down by θ and Γ as described above—must be equal to the CDF of the log-Normal distribution from which κ_{it} is drawn, evaluated at the steady-state cutoff κ^* in (C.32). The second is the restriction that the steady-state value of the partial expectation of κ_{it} implied by the steady state wage and the empirical value of the flow value of non-employment must be equal to the partial expectation of the log-normal distribution, again evaluated at the steady-state cutoff κ^* in (C.32). These two restrictions can be written as

$$F = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\ln(\kappa^*) - \mu_\kappa}{\sigma_\kappa\sqrt{2}}\right)\right] \tag{C.33}$$

$$E[z] - bw = e^{\mu_\kappa + \sigma_\kappa^2/2}\Phi\left(\frac{\mu_\kappa + \sigma_\kappa^2 - \ln(\kappa^*)}{\sigma_\kappa}\right). \tag{C.34}$$

where erf$(\cdot)$ is the Guass error function and $\Phi(\cdot)$ is the CDF of the standard Normal distribution. Given steady-state values of $E[z]$, F, w and κ^*, these two equations can be solved numerically for μ_κ and σ_κ as desired.

[25] See Footnote 14 for an explanation of why we calibrate $\tilde{\eta}$ instead of η.